Modelling Water Jets from a Perforated Tube

Projectile Physics under a Weak–Perspective Camera

`stream_model`

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Abstract

We model the water jets issuing from holes drilled at different heights in a vertical tube. Each jet is a projectile governed by Torricelli's law; the camera views the scene obliquely, so the jets appear in the image as parabolas whose common axis of symmetry is the *projected* gravity direction. We derive the ideal trajectory, show why an oblique (weak–perspective) projection keeps it a parabola, and give a one–parameter–family fit that recovers a single gravity direction, a single Torricelli curvature constant, and a per–jet launch tilt from a few hand–marked traces. The model reproduces the observed jets to a few pixels RMS.

1 The physical setup

A vertical tube is kept full of water; three holes are drilled in its wall at depths $h_1 < h_2 < h_3$ below the free surface. Water exits each hole as a jet and falls into a basin (Figure 1). Deeper holes are under greater pressure, so they squirt faster and reach farther.

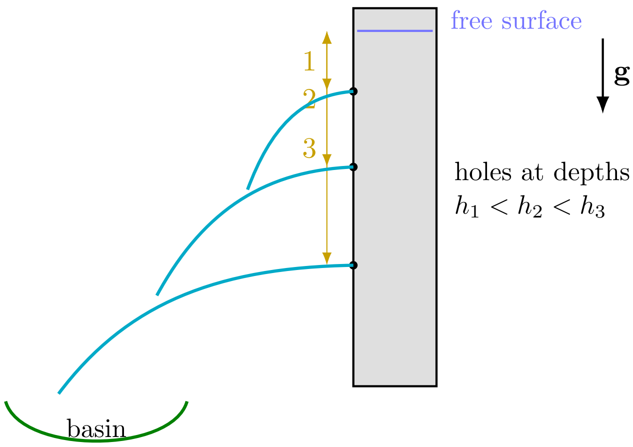

Figure 1: The rig. Each hole emits a parabolic jet; the deepest hole (h_3) has the highest exit speed and the longest range.

2 The jet model

2.1 Exit speed: Torricelli's law

Applying Bernoulli's equation between the free surface (atmospheric pressure, negligible velocity) and a hole at depth h gives the efflux speed

$$v = \sqrt{2\,g\,h}. \tag{1}$$

2.2 Ideal trajectory

Take the hole as origin, x along the (horizontal) exit direction and y pointing down. For an ideal horizontal launch,

$$x(t) = v\,t, \qquad y(t) = \tfrac{1}{2}\,g\,t^2. \tag{2}$$

Eliminating $t = x/v$ and substituting (1),

$$y = \frac{g\,x^2}{2v^2} = \frac{g\,x^2}{2\,(2gh)} = \boxed{\frac{x^2}{4h}}. \tag{3}$$

The gravitational constant cancels: the *shape* depends only on the depth h. A deeper hole (larger h) gives a flatter, longer parabola, recovering the qualitative picture of Figure 1.

Angled launch. Real drilled holes do not all point horizontally, so each jet carries a launch direction $\hat{\mathbf{L}}_i$. The path is still a parabola; only the tangent at the hole changes. We keep this freedom as a per–jet parameter.

3 The projection model

The camera does not look squarely at the plane of the jets: the photograph is oblique. We model the camera as *weak–perspective* (scaled orthographic), i.e. an affine map M from the vertical jet plane Π to the image plane.

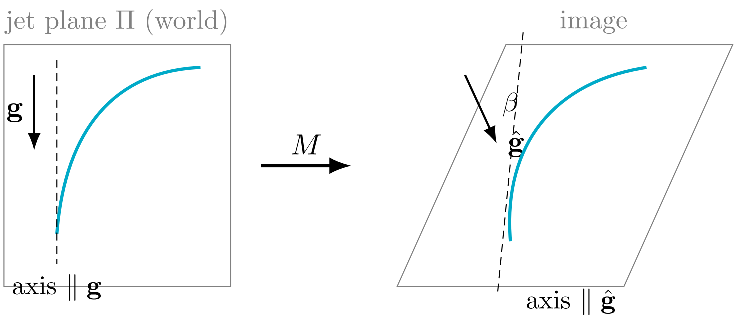

Figure 2: An affine camera maps the world parabola to an image parabola. The axis of symmetry maps to the *projected* gravity direction $\hat{\mathbf{g}}$, tilted by β from image–down. All jets share this single $\hat{\mathbf{g}}$.

3.1 Affine maps preserve parabolas

A parabola is a quadratic curve. An affine map sends quadratics to quadratics and, crucially, sends the parabola's axis of symmetry (parallel to $\mathbf{g}$ in the world) to a fixed image direction

$$\hat{\mathbf{g}} = \frac{M\,\mathbf{g}}{\|M\,\mathbf{g}\|}. \tag{4}$$

Because every jet falls under the *same* gravity, every image jet is a parabola with the *same* axis direction $\hat{\mathbf{g}}$. This shared direction is the backbone of the model.

3.2 Image–space form

Writing $\mathbf{O}_i$ for the image position of hole i, the projected trajectory is

$$\mathbf{p}_i(t) = \mathbf{O}_i + v_i\,t\,\hat{\mathbf{L}}_i + \tfrac{1}{2}\,a\,t^2\,\hat{\mathbf{g}}, \tag{5}$$

where a is the image–space gravitational magnitude (a camera/scale constant) and $\hat{\mathbf{L}}_i$ the projected launch direction.

3.3 Gravity–aligned coordinates

Parameterise the gravity angle β and define the orthonormal pair

$$\hat{\mathbf{g}}(\beta) = (\sin\beta,\ \cos\beta), \qquad \hat{\mathbf{e}}(\beta) = (\cos\beta,\ -\sin\beta), \tag{6}$$

($\hat{\mathbf{g}}$ is image–down at $\beta = 0$). For an image point $\mathbf{p}$ let $u = \mathbf{p} \cdot \hat{\mathbf{e}}$ (across) and $w = \mathbf{p} \cdot \hat{\mathbf{g}}$ (along gravity). In these coordinates (5) becomes an ordinary upward parabola,

$$w = A\,u^2 + B\,u + C, \tag{7}$$

because the only quadratic term, $\frac{1}{2}at^2$, lies wholly along $\hat{\mathbf{g}}$.

3.4 Torricelli ties curvature to depth

The curvature in (7) satisfies $A_i \propto a/v_i^2$. Combining with Torricelli $v_i^2 = 2gh_i$ gives the central constraint

$$\boxed{A_i = \frac{K}{h_i}}, \qquad K \text{ shared by all jets.} \tag{8}$$

So the three free curvatures collapse to a single constant K and the known depths h_i. Torricelli fixes the speed *magnitude*; the drilled direction stays free, carried by the per–jet linear term B_i (the launch tilt).

4 Fitting the model

Given hand–marked jet traces $\{(u, w)\}_i$, hole positions $\mathbf{O}_i$ and depths h_i, we fit

$$\min_{\beta,\,K,\,\{B_i\}} \sum_i \sum_{(u,w)\in\text{trace}_i} \Big(w - w_{h_i} - \tfrac{K}{h_i}\,(u^2 - u_{h_i}^2) - B_i\,(u - u_{h_i}) \Big)^2, \tag{9}$$

where (u_{h_i}, w_{h_i}) is the hole in gravity coordinates; the constant C_i is eliminated by forcing each parabola through its hole. For a fixed β the problem is *linear* in K and $\{B_i\}$ (one shared column, one column per jet), solved by least squares. We therefore sweep β on a 1–D grid, refine around the minimum, and read off the parameters.

5 The depth arrows: tube axis $\neq$ jet gravity

The yellow depth markers measure "distance from the top of the tube to each hole". They must run along the *physical tube*, whose image axis we measure directly ($\approx$ 2–3° of lean). This is *not* the projected jet gravity $\hat{\mathbf{g}}$ ($\approx 24°$): perspective foreshortening tilts the apparent parabola axis far more than the tube. Using $\hat{\mathbf{g}}$ for the arrows over–tilts them; the arrows therefore follow the measured tube axis, shifted onto the silhouette edge.

6 Results

Fitting (9) to three marked traces in each view gives the parameters of Table 1. The recovered gravity direction is essentially the same for the still and the clip (24.4° vs. 22.6°), as it must be for one physical scene, and the curvature constants order the jets correctly by depth. Figure 3 overlays the model curves on the source media.

Table 1: Fitted model parameters.

	β (grav.)	K	RMS	hole depths (cm)		
view	deg	—	px	h_1	h_2	h_3
photo	24.4	0.256	7.5	4.7	9.7	14.1
video	22.6	0.390	6.6	4.8	9.7	14.5

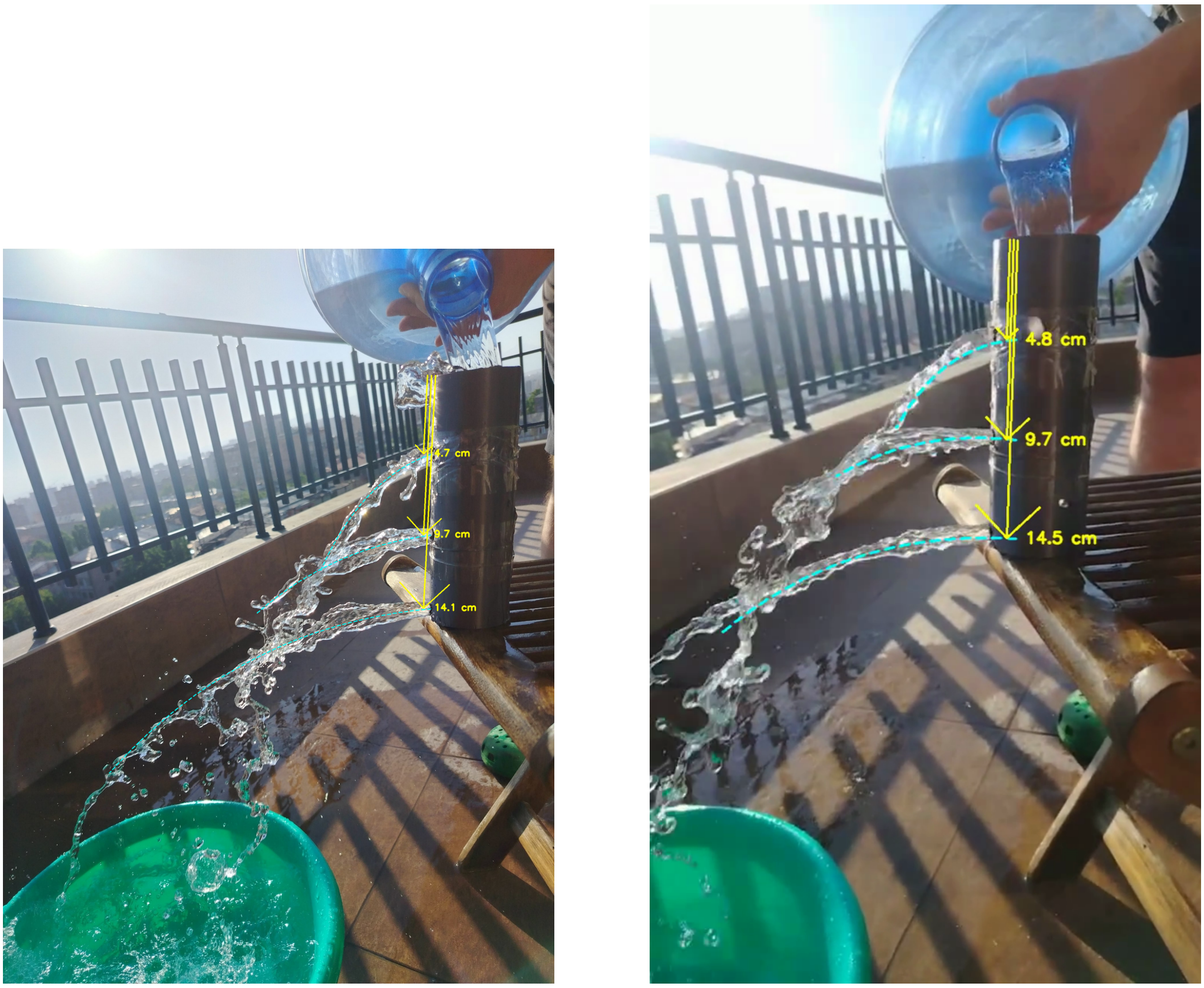

Figure 3: Model curves (cyan, built from the fitted parameters) over the still (left) and a frame of the clip (right). Yellow arrows give the top–to–hole depth in centimetres; they follow the measured tube axis. The cyan parabolas are *generated* by (5)–(8), not traced.

Notation

h_i	hole depth below surface	$\hat{\mathbf{g}}$, $\hat{\mathbf{e}}$	gravity / baseline unit vectors
v_i	exit speed $= \sqrt{2gh_i}$	β	gravity angle from image–down
$\hat{\mathbf{L}}_i$	launch direction	K	shared Torricelli curvature
$\mathbf{O}_i$	hole image position	B_i	per–jet launch tilt